A CONCEPT MAPPING SCORING ALGORITHM BASED ON PROPOSITION CHAINS

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Abstract. Concept maps are widely used in education and have a significant impact on teaching and learning, but it’s really a challenge for teachers and researchers to assess concept maps both efficiently and effectively. Computer-based Concept Mapping Systems (CCMS) have been developed to assess concept maps automatically. However, current scoring methods applied in the CCMS mainly focus on the match of concept nodes, relation links or each separate proposition, paying little attention to the relationship between propositions. In this paper we propose a novel scoring algorithm based on Proposition Chains. Considering concept maps as directed graphs, we define a Proposition Chain as a linked list consisted of all the propositions in one of the longest paths in the graph, thus it can show the relationship among a group of propositions. We aim to solve some problems existing in previous scoring methods and we believe that the proposed scoring algorithm can be easily applied in the CCMS because of its simple storage structure and explicit scoring procedure.

1 Introduction

Based on the learning psychology of David Ausubel, Prof. Joseph D. Novak at Cornell University presented concept map as an instructional technique in the 1980’s. According to Novak (1982), concept maps are tools for organizing and representing knowledge. One of its original uses in education is to assess what a learner already knows (Coffey et. al., 2003). Over the last 20 years, concept maps have been widely used in education.

Since concept map is considered as a great teaching and learning tool, many educators and researchers put emphasis on assessing concept maps. When using paper and pencil, people spent lots of time and energy drawing and assessing concept maps. To cope with this problem, computer-based concept mapping systems (CCMS) have been developed. It should be noted that the scoring methods mentioned in this paper refer to what could be applied in CCMS. Since a computer can not understand semantics of concept maps all by itself, the usual solution is to use a reference map. The similarity value is decided through the comparison of a concept map and a relevant reference map.

A considerable amount of researches have been done during the last few decades and various scoring methods have been proposed (Lomask et al., 1992; McClure & Bell, 1990; Novak & Gowin, 1984; Acton et al., 1994; Goldsmith, Johnson, & Acton, 1991). Actually each scoring method or algorithm has its advantages and limitations. Many researchers also made changes or improvements to these existing scoring methods for their own purposes (Markham, K., Mintzes, J. & Jones, G., 1994; Chen et al., 2003; Chang et al., 2005; Agrawal & Srikant, 1996). Generally speaking, previous methods mainly take concepts nodes, relation links or each separate proposition into consideration. However, they may not take proposition as a whole or lack consideration of hierarchical levels (Chen et. al., 2003); or there may be no explicit instructions on how to score it (Novak & Gowin, 1984), etc. On the basis of previous methods, we present a new algorithm based on Proposition Chains. We aim to solve some problems existing in previous scoring methods and we believe it can be easily applied in the CCMS. This algorithm takes propositions as basic scoring items, and its purpose is to represent and assess the semantic meanings between propositions through the mode of Proposition Chain.

2 Basic Terminologies and Assumptions

We will provide in this section basic terminologies and assumptions which are necessary for the understanding of subsequent algorithm. Concept maps consist of concept nodes and relation links between the concept nodes. According to Novak and Gowin(1984), concept maps are intended to represent meaningful relationships in the form of propositions, which are defined as two concepts connected by a labeled arrow indicating the relationship between the concepts in a semantic unit. If \( C = \{C_1, C_2, ..., C_m\} \) represents the sets of concept nodes, and \( R = \{R_1, R_2, ..., R_n\} \) represents the sets of relation links, we define a concept map as a directed graph \( CM = (C, R) \). If \( c_i, c_j \in C \), \( r_{ij} \in R \), then \( P = (c_i, r_{ij}, c_j) \) represents a proposition in this concept map, and \( r_{ij} \) includes both the relationship and direction of \( c_i \) and \( c_j \).
The assessment discussed in this paper includes reference maps and to-be-assessed maps. A reference map (R-map) is drawn by teachers or domain experts, while a to-be-assessed map (A-map) is usually constructed by students. For a specific concept map task, there is only one R-map with several A-maps. We compare each A-map to the R-map, and then we can get the similarity value of each A-map. In a specific task, there’s always a main topic concept node, which we ask the teachers or experts to mark after they finish the R-map. This main topic concept node is defined as a root node in our comparison procedure.

2.1 Accuracy of Proposition
Ruiz-Primo and Shavelson (1999) state that they favor scoring criteria that focus more on the adequacy of the propositions over those that focus simply on counting the number of map components (e.g. nodes and links). Accuracy of proposition in this paper means the correctness of each separate proposition of the A-maps compared with the R-map. According to the method proposed by McClure and Bell (1990) and the assessment using “weighted concept map” by Chang et al. (2005), we define the accuracy of propositions at three levels: correct, partially correct and incorrect. Let \( CM = (C, R) \) be a R-map and \( CM^S = (C^S, R^S) \) be a A-map. If \( c_i, c_j \in C \) and \( r_{ij} \in R \), then \( P_{t_y} = (c_i, r_{ij}, c_j) \) represents a proposition in \( CM \).

**Situation 1:** If (1) \( c_i \notin C^S \), or (2) \( c_j \notin C^S \), or (3) \( c_i, c_j \in C^S \), but there’s no relation link between them, namely \( P^S_{t_y} = (c_i, r_{ij}, c_j) \) or \( P^S_{t_y} = (c_j, r_{ij}, c_i) \) does not exist in the A-map, we deem that the mapper doesn’t grasp the relationship between concept nodes \( c_i \) and \( c_j \), so that the proposition is incorrect.

**Situation 2:** If \( c_i, c_j \in C^S \) and \( r_{ij} \in R^S \), then \( P^S_{t_y} = (c_i, r_{ij}, c_j) \) or \( P^S_{t_y} = (c_j, r_{ij}, c_i) \) represents a proposition in \( CM^S \).

The following procedure shows how to judge the accuracy of propositions in the basic scoring algorithm based on Proposition Chains. Details in the extended scoring algorithm are discussed in later section.

1. If there is a proposition \( P^S_{t_y} = (c_i, r_{ij}, c_j) \) in the A-map, then: (1) If \( r'_{ij} = r_{ij} \), \( P^S_{t_y} \) is correct; (2) If \( r'_{ij} = \Phi \), \( P^S_{t_y} \) is partially correct (We use \( \Phi \) as the symbol of lacking label of the relation link. similarly hereinafter.);
   (3) If \( r'_{ij} \neq r_{ij} \) (not including \( r'_{ij} = \Phi \)), \( P^S_{t_y} \) is incorrect.

2. If there is a proposition \( P^S_{t_y} = (c_j, r'_{ij}, c_i) \) in the A-map, then: (1) If \( r'_{ij} = \Phi \), \( P^S_{t_y} \) is partially correct; (2) If \( r'_{ij} \neq \Phi \), \( P^S_{t_y} \) is incorrect.

2.2 Proposition Chains
According to Novak (1984), the scoring method contains four components, including propositions, hierarchies, cross-links and examples. However, Novak also claims that it may appear disturbing to see that the same sets of concepts can be represented in two or more valid hierarchies. When the concept map is organized in a network structure or the hierarchy structure isn’t obvious, we can hardly distinguish the hierarchies of a map or decide whether the relation links are cross-links or not.

Considering concept map as a directed graph, we define a Proposition Chain as a linked list consisting of all the propositions in one of the longest paths in the graph, thus it can show the relationships among a group of propositions. Proposition Chains integrate concept nodes, relation links, hierarchies and semantics altogether and imply the hierarchy structure from generalization to specification. Each Proposition Chain can be translated into a sentence with continuous semantic understanding between concepts. A concept map can be taken apart into several Proposition Chains. The R-map shown in Figure 1 includes six Proposition Chains in it. For example, it forms a Proposition Chain from concept nodes A to K, so \( PC_1 = \{(A, R_{AB}, B), (B, R_{BD}, D), (D, R_{DG}, G), (G, \text{e.g.}, K)\} \) is a Proposition Chain with four propositions.
2.3 Discontinuity of a Proposition Chain

Compared with a Proposition Chain in the R-map, discontinuity of this Proposition Chain may occur in an A-map. We define the discontinuity of a Proposition Chain in an A-map as that there is no relation link between two certain concept nodes or the accuracy of certain propositions is incorrect compared to the relevant Proposition Chain in the R-map. As shown in Figure 2, the reference Proposition Chain (a) embodies three propositions. As there’s no relation link between concept nodes B and C in A-map (b), we define the Proposition Chain discontinued once. The discontinuity of a Proposition Chain signifies the interruption of semantics, thus points should be subtracted.

It is noted that there’s an exception. As in Figure 2, the A-map (b) and (c) each form only two propositions comparing to R-map (a). When the discontinuity occurs at the end of the Proposition Chain such as A-map (c), the first two propositions still seem to express a continuous meaning. Therefore, the situation with discontinuity at the start or the end of a Proposition Chain doesn’t count.

2.4 Leap Relationship

Only in the extended algorithm we take leap relationship into consideration, because it may refer to the semantic match of concepts. A proposition appears in an A-map may not exist in an R-map. However, if the concept nodes $c_i$ and $c_j$, which form a proposition $P_{rij} = (c_i, rij, c_j)$ in the A-map, also exist in the R-map but connect indirectly, then we call the relation link $rij$ in the A-map as a leap relationship. We define $F_{rij}$ as leap value, and it shows the hierarchy level between the two concept nodes in the R-map. We should mark all the concept nodes in a Proposition Chain. The start node of a Proposition Chain is marked as level 1, and other nodes is marked as their own hierarchy levels $H_{ci}$ by increasing numbers. Thus, the leap value $F_{rij}$ between concept nodes $c_i$ and $c_j$ is computed as follows

\[ F_{rij} = H_{cj} - H_{ci}. \]

We still take reference Proposition Chain (a) in Figure 2 as an example. Figure 3 shows a to-be-assessed Proposition Chain and concept nodes B and D connect directly without the concept node C existing in the reference Proposition Chain. Then relation link $R_{BD}$ is a leap relationship. We mark concept node B as level 2, and D as level 4 in the reference Proposition Chain. According to the formula described above, the leap value $F_{R_{BD}} = 4 - 2 = 2$. 

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**Figure 1.** An example of a R-map and a A-map

**Figure 2.** Diagram of discontinuity of a Proposition Chain

**Figure 3.** A to-be-assessed Proposition Chain with a leap relationship between B and D.
3 Scoring algorithm based on Proposition Chains

A concept map can be constructed with created linking phrases (C) or with selected linking phrases (S). Yue Yin et al. (2005) conclude that the C technique is better than the S technique in capturing students’ partial knowledge, even S technique can be scored more efficiently than C. Since a computer can not understand the semantics of concept maps all by itself, we divide the scoring algorithm into two categories: the basic one and the extended one. When constructing a map with selected phrases, we choose basic algorithm. When constructing a map with created linking phrases, we should consider the factors mentioned in the extended algorithm. As Debbie (2004) concludes, comparisons between an R-map and A-maps are a relatively new area of research and require an algorithm and computer program to quantify the maps. The scoring algorithm based on Proposition Chains will be described in detail as the following.

3.1 Basic Scoring Algorithm Based on Proposition Chain

Step 1, traverse the R-map, collect the sets of reference Proposition Chain, and calculate the total score of R-map. This step is only taken in the R-map. We define the root node marked by teachers or experts as search start. Using Deep-First-Search algorithm, we start searching from the root node to other nodes along the outgoing arc. When visiting a visited node or the node’s outgoing arc is minimized (usually zero), the search stops and the longest proposition sets formed during the search is called a Proposition Chain. In the case that the search stops when reaching a visited node, this node becomes the last node of this proposition chain. We collect all the Proposition Chains existing in an R-map by back-tracking algorithm, and form a set of Proposition Chain \( \{ PC_n \} \). During the search process, we should record some important information and calculate some necessary data.

- **Sets of Proposition Chain \( \{ PC_n \} \).** Concept nodes and relation links are important elements in the Proposition Chains when matching. We should calculate the number of propositions \( N(PC_n) \) in every Proposition Chain.
- **Hierarchy Level of Each Concept Node \( H_c \).** The hierarchy level of each concept node in every Proposition Chain should be marked, as the same concept node may appear in different Proposition Chains at a different hierarchy level.
- **Score of R-map \( S(PC_n) \).** Assume each proposition in a R-map values 1 point, then the total score of a R-map is computed as follows:

\[
S(PC_n) = \sum_{n=1}^{N} N(PC_n).
\]

Step 2, traverse the A-map, collect the sets of to-be-assessed propositions, compare each proposition with the R-map, and calculate the total score of the A-map. This step is only taken in the A-map. We need to compare each proposition in the A-map with the sets of Proposition Chains in the R-map. During the match process, the accuracy of propositions and discontinuity of a Proposition Chain should be taken into consideration.

- **Sets of To-be-assessed Propositions \( \{ P_{tq} \} \).** Traverse the A-map, and store all propositions in detail (including both concept nodes and relation links), then forms the sets of propositions \( \{ P_{tq} \} \).
- **Match Score of Proposition \( S(PC_{tq}) \).** Compare a proposition in a A-map with all the Proposition Chains in the R-map, and make a mark of the Proposition Chain in R-map when they match. Assuming that the accuracy of proposition value is \( A_{pc} \), we define \( Ap_{tq} \) as 1 point when proposition is correct, 0.5 point when it’s partially
correct, and 0 point when it’s incorrect value $A_{\text{rji}} = 0$. According to the accuracy of each proposition, we can add up the scores of all matches, then the total score of a Proposition Chain is:

$$S(\text{PC}^n) = \sum_{P^s_{ijr} \in \{\text{PC}_n^s\}} A_{P^s_{ijr}}$$

- **Score of Discontinuity in a Proposition Chain** $S(\text{D}_{\text{PC}_n^s})$. According to the sets of propositions $\{P^s_{ijr}\}$ in an A-map, we can count the discontinuity numbers of a Proposition Chain compared with an R-map. Assume discontinuity in a Proposition Chain is computed as follows:

$$S(\text{D}_{\text{PC}_n^s}) = \frac{N(\text{D}_{\text{PC}_n^s})}{N(\text{PC}_n^s)}$$

**Step 3, calculate the similarity value with comparison of R-map and A-map.** Bring the scores of the above two steps into formula, and the similarity value is computed as follows:

$$\alpha = \frac{S(\text{PC}^n) - S(\text{D}_{\text{PC}_n^s})}{S(\text{PC}_n^s)}$$

### 3.2 Extended scoring algorithm based on Proposition Chain

The extended algorithm is presented to score concept maps constructed with created phrases. The difference between the basic and extended algorithm is how to judge the accuracy of a proposition and deal with the additional propositions. Moreover, the emphasis is placed on the semantic match.

**3.2.1 Accuracy of propositions**

The judgment of the accuracy of proposition is just the same as what we presented above, except for a minor change in situation 2. If there is a proposition $P^s_{ijr} = (c_j, r_{ij}, c_i)$ in the A-map, then: (1) If $r_{ij} = r_{ji}$, $P^s_{ijr}$ is considered to be correct because $r_{ji}$ has the opposite semantic meaning; (2) If $r_{ij} = \emptyset$, $P^s_{ijr}$ is partially correct; (3) If $r_{ij} \neq r_{ji}$ (not including $r_{ij} = r_{ji}$), $P^s_{ijr}$ is incorrect.

**3.2.2 Sets of additional propositions**

We record the propositions in an A-map but not in the R-map, and they form the sets of additional propositions $\{P^s_{\text{add}}\}$. The leap relationship is among the additional propositions. If the leap relationship is correct, we should add some reasonable points to the total score, which we call the correct score of leap relationships $S(L^s_{r_{ij}})$. Whether the leap relationship is correct or not depends on the types of the relation links and the transitivity of the link words. Based on the previous research, one may classify the type of the knowledge into the following items: (1) classification relation, (2) successive relation, (3) belonging relation, (4) causative relation, and (5) composition relation (SHEN Rui-min et. al., 2002). It can also be categorized into: (1) belonging relation, (2) characteristic relation, (3) causative relation, (4) functional relation, (5) explanation relation, (6) definition relation, (7) example relation, and (8) type relation, etc (Novak & Gowin; Holley, Dansereau, McDonald, Garland & Collins). If the type of relation links is appropriate, then we can put them in certain sequences to transfer the semantic meaning of propositions. We cannot describe the detailed information here as it is a complex problem to explain when and how it can transfer the semantic meanings.

In **Figure 4**, for example, the R-map (a) shows a Proposition Chain with two propositions including (living thing, include, animals) and (animals, include, vertebrates). If only one proposition (living thing, include, vertebrates) appears in the A-maps, we consider it as partially correct, for relation link word “include” can somewhat transfer the semantics.
To show the influence on Proposition Chains, we define the correct leap relationship index as $\frac{1}{L_{ij}}$. In terms of student’s understanding, the more intervals there are between hierarchies, the more obscure cognitive recognition it reflects, and the fewer score should be given. So the score of corrected leap relationship should be calculated as follows:

$$S(L^s_{ij}) = \sum \frac{1}{L_{ij}}$$

Except for the leap relationships, other additional propositions existing should be assessed by human judgment. We record them as feedback information to students but do not deal with them.

### 3.2.3 Similarity value of the A-map

The method is similar to that in the basic scoring algorithm, but it requires consideration of considering two more factors mentioned in the sections above. Bring all the scores into the formula as follows:

$$\alpha = \frac{S(PC_n^s) - S(D_{PC_n^s}) + S(L^s_{ij})}{S(PC_n)}.$$  

### 4 Example analysis

Since constructing a map with selected phrases can be more easily assessed, the following example is only for the purpose of illustrating the procedure of extended algorithm. An example of concept map is shown in Figure 1. Note that: (1) the identical symbols in the R-map and the A-map indicate complete consistency. (2) The relation $R'_{DH}$ in the A-map between nodes $D$ and $H$ is different from that in the R-map. (3) There’s no relation link label between concept nodes $C$ and $F$ in the A-map. (4) Both the direction and semantic of relation link label between concept nodes $C$ and $F$ in the A-map is opposite to that in the R-map. Applying the scoring algorithm based on Proposition Chains to the example, we can get the following score.

**Step 1**, traverses the R-map, collects the sets of Proposition Chains, and calculate the number of propositions in each Proposition Chain as follows:

$$PC_1 = \{(A, R_{AB}, B), (B, R_{BD}, D), (D, R_{DG}, G), (G, R_{GK}, K)\},$$

$$PC_2 = \{(A, R_{AB}, B), (B, R_{BD}, D), (D, R_{DG}, H), (H, R_{HL}, L)\},$$

$$PC_3 = \{(A, R_{AB}, B), (B, R_{BD}, D), (D, R_{DG}, H), (H, R_{HC}, C), (C, R_{CE}, E), (E, R_{EI}, I)\},$$

$$PC_4 = \{(A, R_{AB}, B), (B, R_{BD}, D), (D, R_{DG}, H), (H, R_{HC}, C), (C, R_{CE}, F), (F, R_{FJ}, J)\},$$

$$PC_5 = \{(A, R_{AC}, C), (C, R_{CE}, E), (E, R_{EI}, I)\},$$

$$PC_6 = \{(A, R_{AC}, C), (C, R_{CF}, F), (E, R_{FJ}, J)\}.$$  

$$N(\text{n}) = N(\text{PC}_1) + N(\text{PC}_2) + N(\text{PC}_3) + N(\text{PC}_4) + N(\text{PC}_5) = 4 + 4 + 6 + 6 + 3 + 3 = 23.$$  

**Step 2**, traverse the A-map, and make marks in the Proposition Chains as above. The mark is described as: (1) the straight underline represents correct propositions and scores 1 point, (2) the dashed underline represents partially
correct propositions and scores 0.5 point, and (3) the unmarked proposition is incorrect or missing and scores 0 point. Then the match score and discontinuity scores of each proposition is as follows:

\[
S(s_{1PC})=3, S(s_{2PC})=3, S(s_{3PC})=4, S(s_{4PC})=4.5, S(s_{5PC})=2, S(s_{6PC})=2.5.
\]

\[
N(D_{PC1})=N(D_{PC2})=N(D_{PC3})=N(D_{PC4})=1, N(D_{PC5})=N(D_{PC6})=0.
\]

\[
S(D_{PC1})=S(D_{PC2})=\frac{1}{4}, S(D_{PC3})=S(D_{PC4})=\frac{1}{6}, S(D_{PC5})=S(D_{PC6})=0.
\]

The set of additional propositions \( P^\text{add}_\text{add} \) consist of \((M, RMA, A)\) and \((A, RAG, G)\). \( RAG \) is shown to be a leap relationship, and the hierarchy level value in Proposition Chain \( PC_1 \) is \( \lambda_{v1} = H_{c1} - H_{c1} = 4-1 = 3 \). Assuming that \( RAG \) indicates reasonable semantics, the score of correct leap relationship \( S(F_{ij}^s) = 1/3 \). As \( RMA \) appears to be an additional relation link, we just record it for a feedback. The proposition \((E, REI, I)\) is missing, and we record it for a feedback as well. Then, we can get the similarity value of this A-map as follows:

\[
\alpha = \frac{(3 + 3 + 4 + 4.5 + 2 + 2.5) - (1/4 + 1/4 + 1/6 + 1/6) + 1/3}{23} = 0.8043.
\]

5 Discussion and Future Work

The most important function of concept map is to facilitate the learners’ thinking, comprehension and recollection through the process of concept mapping. The educators score the concept map in order to do quantitative analysis, or compare the students’ pre and post learning effects. Some important information will absolutely be lost if only considering quantitative assessment. Therefore, we should explain the score scientifically concluded by the scoring algorithm of Proposition Chains. It is important to point out that the similarity value only shows the exactitude rate of a A-map compared with a R-map.

The scoring methods for concept mapping are established step by step with modification and will never set a standard criterion. According to Novak and Gowin (1984), any scoring key, as well as the numerical scores employed to score concept maps, is somewhat arbitrary and subjective. He encourages educators to experiment with different values. Few of the published scoring methods can be applied to the CCMS, especially when constructing a map with created phrases. We attempt to integrate useful parts of previous scoring methods into the proposed algorithm, and try to solve the problems existing in them. Theoretically, the advantages of applying the scoring algorithm based on Proposition Chains to the CCMS are: (1) clear scoring steps, (2) standardized scoring rules, (3) simple storage structure. In addition, Chinese concept map software “Easy Thinking-Cognitive Assistant” is under development by Institute of Knowledge Science & Engineer in BNU, and we will choose this new scoring algorithm as its basic scoring rule.

We have just started our research in automated assessment of concept maps, and both theoretical and practical problems that are waiting forward need to be tackled at the moment. Just like the research done by Scott et al. (2004) in an online concept mapping tool “C-TOOLS”, the percentage of automated-assessed propositions is still low with the help of software “Robograde\textsuperscript{TM} and WordNet\textsuperscript{TM}”. The CCMS need a huge word database as the support of semantic matches as well. At the same time, network supported collaborative concept mapping is popularized so fast that it becomes an important future research to design a criterion to score the work of individuals or groups. Since we haven’t applied the proposed algorithm to real-world practice, its reliability and validity still need further evidence. Considerable more work needs to be done in this area.

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